Transformation of a Polynomial for a Contraction Wall Profile.

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In the design of flow-conditioning contractions for low-speed (<50m/s) wind tunnels, several desirable characteristics of the wall profile are identified, including: A wall profile having zero first and second derivatives, and inlet and outlet profile radii roughly proportional to the area, that is, the inlet radius is greater than the outlet radius. The result is hoped to be the most favorable combination of flow uniformity, thin boundary layers, and negligible losses.

A fifth-order polynomial can be readily developed, as by Bell and Mehta (1988), satisfying the condition of zero first and second derivatives at the inlet and outlet. The polynomial Bell and Mehta originally developed has been repeatedly employed successfully in contraction design in two and three dimensions:

Normalizing for length,

$$x = \frac{X}{L} \quad (1)$$

$$h = \left[-10(x)^3 + 15(x)^4 - 6(x)^5\right](H_i - H_O) + H_i \quad (2)$$

where H_i and H_o are the height of the contraction wall, from datum at the axis of symmetry, at the inlet and outlet respectively.

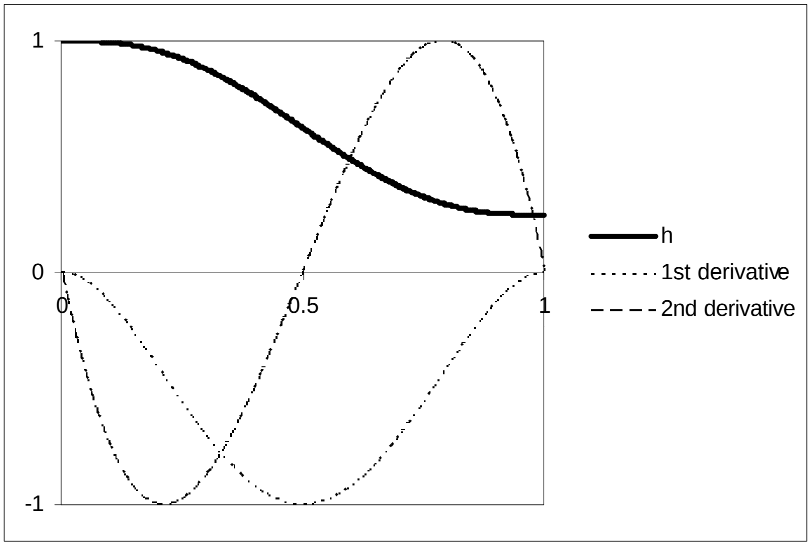

Figure 1: Bell and Mehta's Fifth-order Polynomial, with First and Second Derivatives

Equation (2) has proven so successful that this transfer function has largely become the design standard. Irrespective of this, many designers of wind tunnel contractions believe that a more qualitative, experiential design synthesis of contraction profiles, so called “by eye design”, will produce better results. This may be due to Eq.(2) being symmetrical, with identical radii at the inlet and outlet, an arrangement traditionally considered undesirable (Mehta & Bradshaw, 1979).

Using a wall profile described by a function is inherently useful, as it facilitates the creation of data tables to describe cutting templates, tool paths, and the building of jigs for construction.

It would be desirable to develop a function such that any curve characteristics selected by a designer can be described by transformations of (2). It must maintain zero first and second derivatives at the inlet and outlet, and must have continuous first and second derivatives throughout. It is important that this be achieved as simply as possible, so as to be of maximum utility to the designer. Systematic manipulation of as few parameters as possible should produce the curve desired.

The transfer function described herein is an attempt to provide an easy, systematic procedure allowing Bell and Mehta’s fifth-order polynomial to be fitted to a “by eye” curve selected or engendered by the designer of the wind tunnel contraction, satisfying the criteria above.

As it appears unnecessary to re-invent the wheel, the central fifth order polynomial of Bell and Mehta is the function chosen for transformation. Defining Bell and Mehta's core polynomial as η;

$$h = 10x^{3} - 15x^{4} + 6x^{5} \qquad (3)$$

where ξ is dimensionless axial distance measured from the inlet, with ξ=1 at the outlet. This polynomial has been defined as varying between 0 and −1, in order to present the resulting contraction profiles consistently and intuitively. In any attempt to bias a function whose numerical value varies smoothly between zero and one, the simple application of an exponent comes to mind. Simply taking the square root of |η| will provide the profile shown in figure 2:

$$h = 1 - (h)^{\frac{1}{2}} \qquad (4)$$

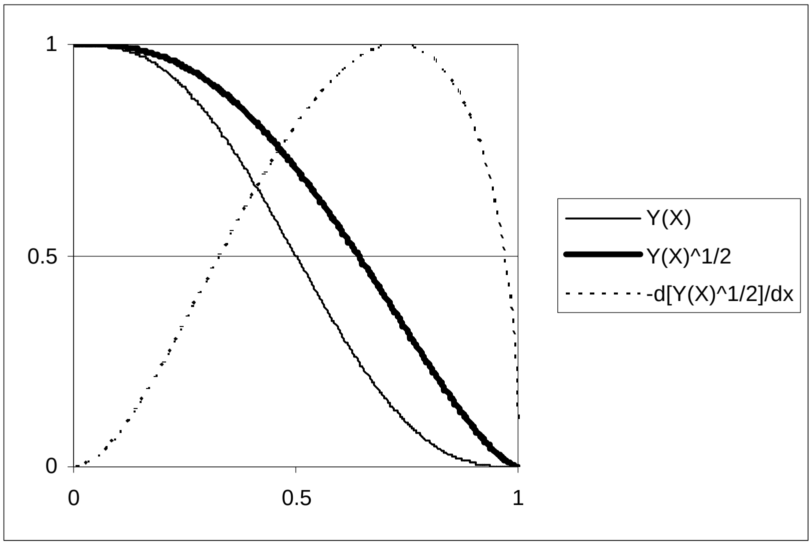

Figure 2: Bell and Mehta’s polynomial and its square root.

At first sight, this simple transformation seems to provide for the desired larger inlet radius than outlet radius. However the radius of curvature at the outlet is too small

Using a transfer function like (2) to transform Bell and Mehta’s polynomial to arbitrary inlet and outlet heights, while incorporating the change in shape provided by raising the polynomial to a power less than unity, , the following transfer function is proposed:

$$h = \left[h \left(H_i^{\frac{1}{a}} - H_o^{\frac{1}{a}} \right) + H_i^{\frac{1}{a}} \right]^a \qquad (5)$$

where α is some function of ξ, defined for 0<ξ<1. This function will be referred to as the alpha function throughout. (5) is obviously very similar to Bell and Mehta's transfer function. It is the functional nature of α which provides interesting results.

It can be shown that any function α chosen for use in (5), normalized to vary between 0 and 1, will result in a smooth function (5), maintaining first and second derivatives equal to zero at the inlet and outlet of the resulting contraction profile.

In order to demonstrate the effect of α on *h* some sample alpha functions are presented.

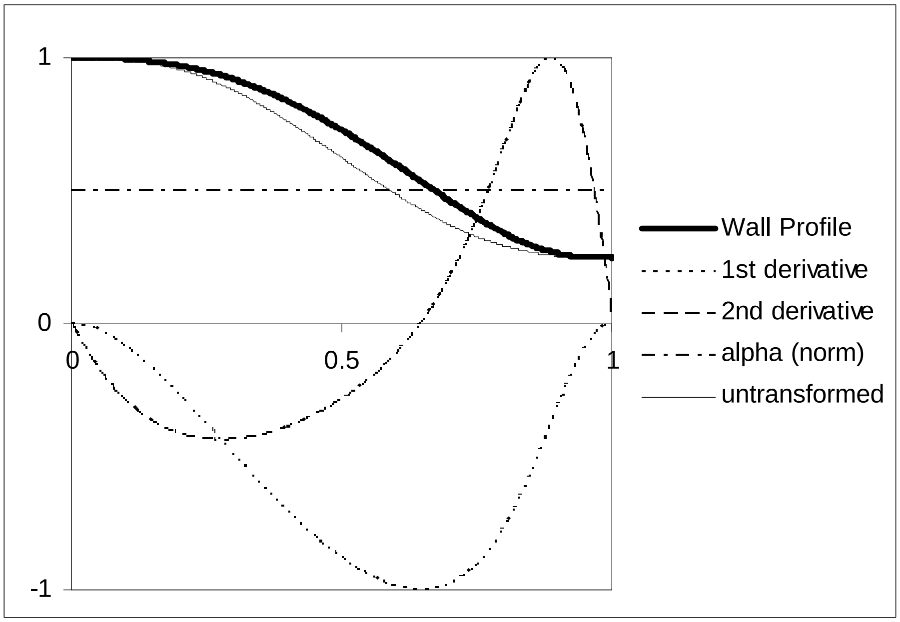

Figure 3: Wall profile, first and second derivatives for α=0.5

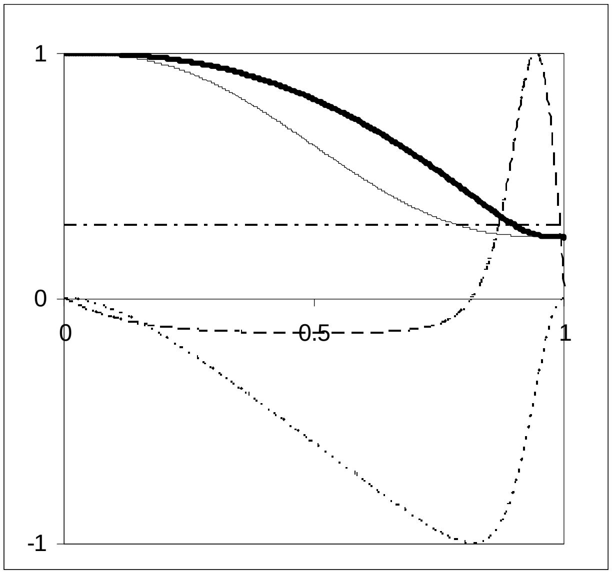

Figure 4: α=0.3

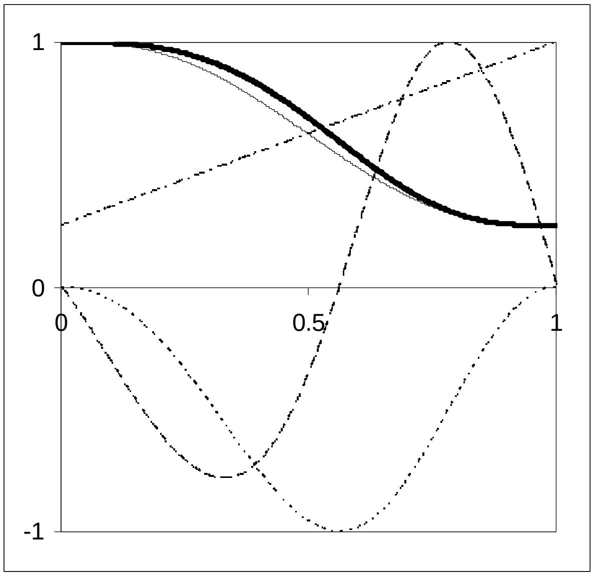

Figure 5: α varies linearly with ξ.

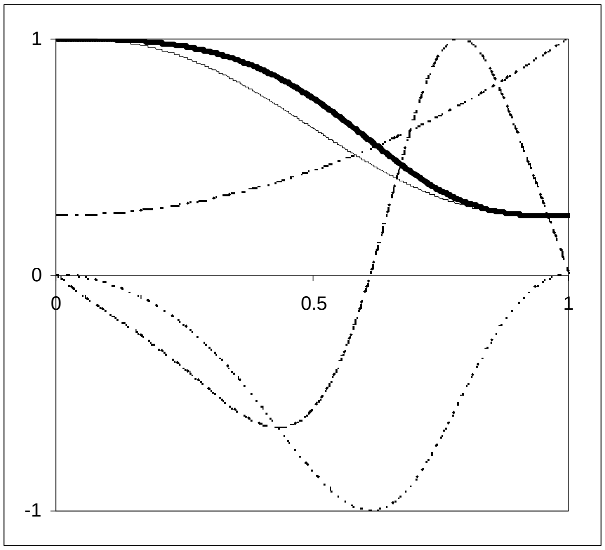

Figure 6: α varies quadratically with ξ.

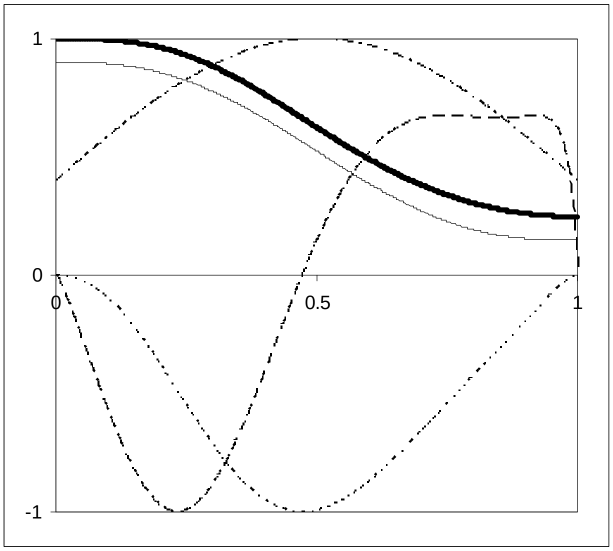

Figure 7: α varies as sinξ – un-transformed curve displaced for clarity.

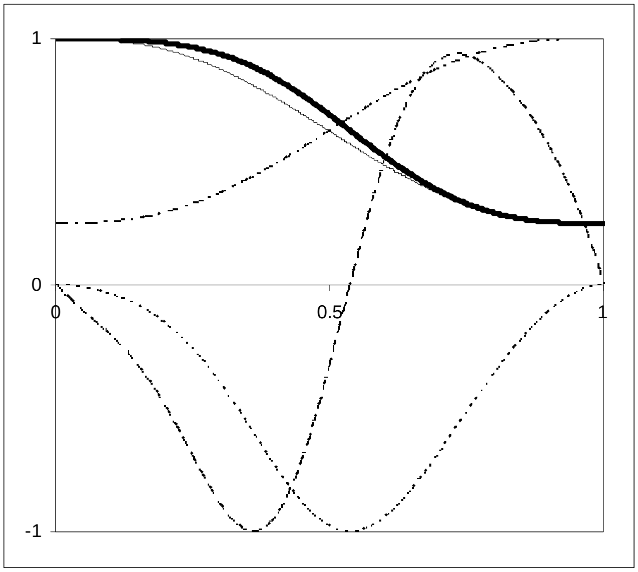

Figure 8: α varies as a fifth-order order polynomial, similar to the η function itself.

It can be seen by inspection of figures 3 and 4 that the wall profile is distorted similar to figure 2. Choosing different values of α can provide a wide range of profiles for a designer to choose. Extending this to selecting α as a function of ξ rather than a constant, it can be argued that favorable wall characteristics, such as a larger radius at the inlet than at the outlet, can be generated by judicious selection of α either as a constant or as a function of ξ.

For instance, if a designer wishes to generate a large radius at the inlet, a smaller radius at the outlet while maintaining a longer transition of the outlet radius to the test section, a quadratic alpha function could be chosen, as in figure 5.

Where a researcher wishes to design a curve using an entirely intuitive technique, this transfer function generation method may prove useful.

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